GPPS-2017-0032

Acoustic Eigenmode Analysis for Swirling Flows in Ducts

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ABSTRACT

The acoustic eigenmodes for swirling flows in ducts are investigated by solving a generalized eigenvalue problem (GEP) for a linearized three-dimensional Euler equation. The mean flow is assumed to be axially and circumferentially uniform but could vary radially with a general swirling. The linearized equation is discretized by using a Chebyshev pseudo spectral method. This method is validated against analytical solutions and is applied to compute eigenmodes at NASA Rotor 67 exit boundary. This work also shows a good promise for a nonreflecting boundary condition (NRBC) construction and tonal noise prediction.

INTRODUCTION

The linear CFD analysis method has been widely used in turbomachinery industry field for tonal noise, flutter and forced response problems prediction because of compromising between accuracy and turnaround time. Before solving a linear problem, one has to compute a nonlinear steady flow solution. Next, a small perturbation assumption is made to derive a set of linearized equations either in a continuous or a discrete manner. As the temporal derivative term becomes a source term in the frequency domain, the linearized equations become mathematically “steady” state. At last, a pseudo time term is introduced so that the well-established convergence acceleration techniques could be employed to speed up the convergence rate of linear equations.

The nonreflecting boundary condition plays a role of great importance for a successful aeroelastic and aeroacoustic computation. If not appropriately treated at the inlet and outlet boundary, the small amplitude unsteadiness may be reflected into the computational domain and corrupt the solution. Inspired by Engquist and Majda's wave splitting method (Engquist and Majda, 1977), Giles analyzed the dispersion relation for linearized 1D and 2D Euler equations, derived analytical eigenmodes and proposed NRBCs (Giles, 1990). However, there exists no general solution for non-uniform 3D flows. To overcome this problem, Hall et al. (Hall, 1993) assumed the analytical Fourier mode in the circumferential direction and computed numerical mode in the radial direction for a general 3D inviscid flow so as to construct the mixed analytical and numerical 3D NRBC. This method has been adopted by other authors (Verdon, 2001 and Sharma, 2007). Later on, it was extended for viscous flows (Moinier et al., 2005 and Moinier, et al., 2007). Alternatively, Petrie-Repar proposed to compute both the circumferential and radial mode in a numerical manner and constructed a full numerical 3DNRBC (Petrie-Repar, 2010).

Kousen (Kousen, 1999) and Ali et al. (Ali et al., 2000) made an isentropic flow assumption and performed a normal mode analysis for ducted flows with swirling using a similar technique. Guan and Wang investigated the effects of mean entropy on the acoustic eigenmodes (Guan and Wang, 2007). Giacche et al. used Giles's tool to compute the normal mode and then to extract the tonal noise information for a modern turbofan engine (Giacche, et al., 2011).

The eigenmodes in a uniform axial flow are computed using a Chebyshev pseudo-spectral discretization. The numerical solutions are validated with analytical ones. Then, a free vortex swirl and a rigid body swirl flow eigenmodes are computed and compared with results in the literature. To demonstrate the application for realistic problem, eigenmode analysis is performed for the NASA Rotor 67. One drawback for this technique is the presence of spurious modes because the numerical discretization violates the original dispersion relation. The spurious mode filtering procedure is also described.

A GENERALIZED EIGENVALUE PROBLEM

The time dependent three-dimensional Euler equations can be written in a cylindrical rotating frame of reference as

\[
\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial \theta} + \frac{\partial H}{\partial r} = S
\]  

(1)

where \(x, \theta, r\) are the axial, circumferential, and radial coordinates, respectively, \(U\) is the vector of the conservative variables, \(F, G, H\) are inviscid fluxes, and \(S\) denotes to the source term due to rotation,
In the above equation, \( \rho \) is the density, \( u, v, w \) are the relative velocities in the axial, circumferential, and radial direction, \( p \) is the pressure, \( I \) is the rothalpy, and \( \Omega \) is the rotational speed. The pressure and the rothalpy are defined as

\[ p = (\gamma - 1)[\rho E - \frac{1}{2}\rho (u^2 + v^2 + w^2) + \frac{1}{2}\rho \Omega^2 r^2], \]

\[ I = \frac{\rho E + p}{\rho} = \frac{\gamma}{\gamma - 1} p + \frac{1}{2}(u^2 + v^2 + w^2) - \frac{1}{2} \Omega^2 r^2. \]

It is assumed that the mean flow field is axially and circumferentially uniform. In addition, the unsteady perturbation is assumed to be relatively small compared to the mean flow. Under these assumptions, the perturbation variables in the far field are governed by a linearized Euler equation

\[ \frac{\partial U}{\partial t} + A_r \frac{\partial U}{\partial x} + A_\theta \frac{\partial U}{\partial \theta} + \frac{1}{r} A_r \frac{\partial U}{\partial r} = A_r U_r, \]

where

\[ U_r = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho w I \end{bmatrix}, \]

\[ M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ u & \rho & 0 & 0 & 0 \\ v & 0 & \rho & 0 & 0 \\ w & 0 & 0 & \rho & 0 \\ v - \Omega r & 0 & 0 & 0 & \rho \end{bmatrix}, \]

\[ q^2 \left( \frac{1}{2} \Omega^2 r^2 \right) = \rho u \rho v \rho w \left( \frac{1}{\gamma - 1} \right), \]

\[ \frac{q^2}{2} \left( \frac{1}{2} \Omega^2 r^2 \right) = \rho u \rho v \rho w \left( \frac{1}{\gamma - 1} \right). \]

Because of the flow periodicity in the circumferential direction, the circumferential eigenmodes are Fourier series. Also, assuming the perturbation frequency is \( \omega \), the vector of the primitive variable perturbations can be written as

\[ U_r = \sum \sum \left. u_{n_x} a_{n} \exp (i \omega t + i m \theta + ik \alpha x) \right|_{\alpha = \infty} \rho_r. \]

This results in a generalized eigenvalue problem (GEP), with an unknown eigenvalue for the axial wave number \( k_{n_x} \) and its associated eigenvector for the radial modeshape \( u_{n_x} \). As there is no analytical general solution to this problem, one
has to discretize the above equation and solve it numerically. If defining a difference operator \( D \) as
\[
D u_{mn} = \frac{1}{r} \frac{\partial}{\partial r} (r A_i u_{mn}),
\]
one can rewrite Eq.(4) as
\[
(-\omega \hat{M} - \frac{1}{r} i m \hat{\Delta} + \hat{D} + \hat{\partial} \hat{A}) u_{mn} = \omega^2 \hat{k} \hat{A} u_{mn},
\]
where \( \hat{M}, \hat{A}, \hat{\Delta}, \hat{D}, \hat{\partial} \) matrices are size of \( 5N \times 5N \), with \( N \) is the radial grid node number.

Initially, a second order finite difference is employed to discretize the radial derivative. However, this formulation introduces a great amount of spurious modes because it fails to resolve high wave number modes. Instead, a Chebyshev pseudo spectral difference operator is applied to alleviate the aliasing problems, see the reference (Kousen, 1999) for more details. In addition, a small amount of numerical fourth order dissipation is added to the Eq.(6) system to ensure the smoothness of the modeshape. The dissipation term employs the spectral radius as a scaling factor (Jameson et al., 1981) and takes the form as
\[
e_{ij} (|w| + c)(\Delta r) \left( \frac{d^4 M u_{mn}}{d r^4} \right),
\]
where \( e_{ij} \) is a tunable coefficient. The spectral accuracy of the Chebyshev discretization enables the spurious modes reduced substantially. For inviscid cases, the system is closed by imposing zero radial velocity boundary condition at the both inner and outer walls. The final discretized GEP is then solved by LAPACK (Anderson et al., 1999).

Identification and Sorting Acoustic Modes

Raw outcomes of the GEP include acoustic, entropic, vortical, and spurious modes. Unless low order acoustic modes can be identified from the raw solutions, this method can rarely be used for industrial applications. The identification and sorting procedure works as follows. First, all of the raw eigenmodes are normalized to have a unit \( L_2 \) norm. Next, the normalized eigenvectors are sorted according to the \( |p| \) magnitude. The first 2N modes with largest \( |p| \) magnitude are retained and classified as acoustic modes. Then, the mode travelling direction is determined by comparing the sign of imaginary part of the axial wave number. Consider the wave definition in Eq.(3), the axial wave number is a complex value, \( k_{mn} = k_{mn,0} + k_{mn,i} \). For physical stable modes, \( k_{mn,0} > 0 \) corresponds to downstream travelling modes and \( k_{mn,0} < 0 \) corresponds to upstream travelling modes. For cut on modes, \( k_{mn,0} = 0 \), one can compute the group velocity \( -\frac{\partial \omega}{\partial k} \) to determine the wave travelling direction (Hall et al., 1993). To avoid complex computation, Moinier et al. proposed to perturb the real frequency with a small negative imaginary part, \( \omega = -10^{-5} \omega \), (Moinier et al., 2005). Then the perturbed axial wave number has an imaginary part \( \hat{k} = -\partial k / \partial \omega \).

In this way, one can rely on the sign of the imaginary part to determine the wave travelling direction. However, there exist spurious modes or unstable modes having a negative imaginary part for real turbomachinery problems. To eliminate these undesired modes, another sorting procedure based on the pressure node is performed. Physically, the first acoustic mode has zero pressure node and the second mode has one pressure node, and so forth. If it happens that some vortical modes are misclassified as downstream travelling acoustic modes because of numerical smoothing, the one with a larger pressure magnitude is likely to be a generic acoustic mode.

RESULTS AND DISCUSSION

Axial Uniform Flow

For the validation purpose, a uniform axial flow of \( Ma = 0.5 \) and \( m = 1 \) in a duct of \( r_{inner} / r_{outer} = 0.5 \) is considered. For cases considered in this paper, only 17 radial grid nodes are used except the NASA Rotor 67 case. Analytically, the acoustic modes in an axial uniform flow are Bessel functions. The entropic and vortical wave number for this case is \( -\omega / Ma = -20 \). As shown in Figure 1, the raw eigenvalue map does show a cluster of eigenvalues around -20. However, there are two nearly the same eigenvalues in the raw map for both the first upstream and downstream travelling acoustic mode. The spurious modes could be easily removed by the pressure node sorting procedure because of point-by-point oscillation. The filtered acoustic eigenvalues and eigenmodes are in excellent agreement with analytical solutions (Moinier et al., 2005) as shown in Figure 2 and Figure 3, respectively.

![Figure 1 Raw Eigenmode Map for an Axial Uniform Flow](image-url)
Next, a free vortex swirl flow is superimposed to an axial uniform flow. The mean flow condition is defined as
\[ u = M_x, v = \frac{\Gamma}{r}, w = 0, \rho = 1, p = \frac{1}{\gamma} + \frac{\Gamma^2}{2} \left( \frac{1}{r^2} - 1 \right), \]
where \( M_x \) is the axial Mach number and \( \Gamma \) is the circulation. The density is assumed to be constant and the static pressure is determined according to the radial equilibrium equation. The flow condition, \( M_x = 0.3, \Gamma = 0.2, \omega = 10, m = 2 \) and \( r_{inner} / r_{outer} = 0.4 \), is identical to the one used by Tam and Auriault (Tam and Auriault, 1998).

On the raw eigenvalue map of Figure 4, there is a set of eigenvalues spreading from -32.9 to -23.4 nearly on the real axis. These are "nearly" or "purely" convected modes. The analytical wave numbers from two-dimensional linearized Euler equations, \( \omega + \frac{M_x m}{r} \), yields the range from -32.0 to -25.0. The eigenvalues labeled by 1 and 2 are beyond the analytical range and are classified as spurious modes because of the point-by-point oscillation modeshape. Even worse, there are ten eigemodes with a negative imaginary part in that set. The eigenvalues labeled with 3 and 4 are classified as spurious acoustic modes also because of point-by-point oscillation modeshape. The filtered first ten upstream and downstream travelling acoustic wave numbers compare well against Tam's results (Tam and Auriault, 1998). To investigate the influence of swirling flow on the acoustic mode, the analysis is performed without the swirling component. As shown in Figure 6, the free vortex swirl flow makes a stronger effect on the low order modes than the high order ones.
Rigid Body Swirl Flow

As a third case, a rigid body swirl flow is superimposed to an axial uniform flow. The mean flow condition is defined as

\[
\begin{align*}
\mathbf{u} &= M_x, v = \Omega r, w = 0, \rho = 1, p = \\
&= \frac{1}{\gamma} \left[ \frac{\Omega^2}{2} \left( 1 - \frac{1}{r^2} \right) \right],
\end{align*}
\]

with \( M_x = 0.3, \Omega = 0.5, \omega = 10, m = 2, r_{inner}, r_{outer} = 0.4 \).

As shown in Figure 7, the filtered eigenvalue compares well with Tam's solutions (Tam and Auriault, 1998). And Figure 8 shows that the rigid body swirl also makes a stronger influence on the low order modes.

NASA Rotor 67 Exit Flow

Having validated this method against simple flow models, eigenmodes at the NASA Rotor 67 exit boundary is investigated. The black circles in Figure 9 represent the steady viscous radial profile obtained by NUMECA/FINE Turbo. However, direct usage of this profile would generate wiggles and challenge the filtering and sorting procedure. To circumvent the problems, the boundary layer is removed, and the rest profile is fitted into a linear function on a Chebyshev-Gauss-Lobatto grid as blue squares. With this ad hoc fix, the computed first four eigenmodes become smooth as shown in Figure 10 and the sorting procedure works without difficulty.
CONCLUSIONS

Acoustic eigenmode analysis for swirling flows in ducts are performed in this paper. The three-dimensional linearized Euler equation is discretized by the Chebyshev pseudo-spectral method. A filtering and sorting procedure to identify the desired low order acoustic modes is emphasized. The accuracy of this method is validated with analytical solutions and other numerical results in the literature. The method shows a good promise for construction a non-reflecting boundary condition and tone noise prediction.

ACKNOWLEDGMENTS

The author would like to thank the AECC CAE Co., LTD. for the permission to publish this work. The author also appreciates the discussion with Dr. Ekici at University of Tennessee, Knoxville on the numerical dissipation model and the help from Dr. X.F., Sun at Beihang university and Dr. J.Z., Feng at AECC CAE Co., LTD's on the application to the NASA Rotor 67 case.

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